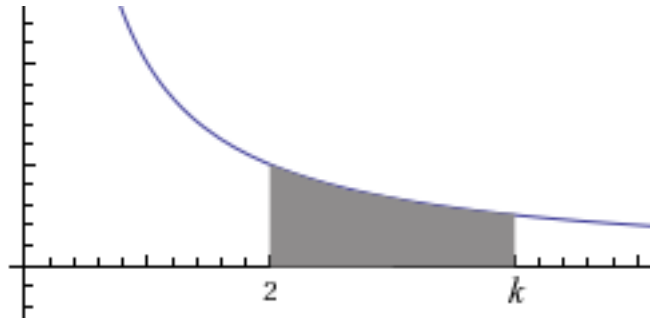


MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2011 MATH CONTEST
SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

1. A club has 15 members. In how many ways can 4 officers consisting of a president, vice-president, secretary and treasurer be chosen?
(A) 54 (B) 60 (C) 32760 (D) 50625
2. From a box containing 4 white, 3 yellow and 1 green ball, two balls are drawn one at a time with no replacement. What is the probability that 1 yellow and 1 white ball are drawn?
(A) 3/14 (B) 9/196 (C) 3/7 (D) 4/7
3. A ball is dropped from a height of 1m and bounces to a height of 2/3 of the previous height. If the ball only moves vertically, how far will it have travelled by the time it stops bouncing?
(A) 3 (B) 5 (C) 5/3 (D) 3/2
4. Let $f(x) = \tan x$, and $g(x) = x^2$. At what value of x in the interval $0 \leq x \leq \pi$ do the graphs of f and g have parallel tangent lines?
(A) 0 (B) 0.660 (C) 2.083 (D) 2.194

5. A portion of the function $f(x) = 1/x$ is plotted at right. For what value of k will the area of the shaded region be $\ln(4)$?
(A) 4
(B) 8
(C) e
(D) e^2



6. If f is a function such that $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?
(A) $\lim_{x \rightarrow a} f(x)$ does not exist (B) $f(a)$ does not exist (C) $f'(a) = 0$ (D) $f(x)$ is continuous at $x = 0$.
7. At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where y is the number of bacteria present and k is a constant. The initial population is 1,500 and the population quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days?
(A) 4 (B) 5 (C) 6 (D) 8
8. Given a triangle, where a and b are sides of the triangle and θ is the angle between those sides, what angle θ will maximize the area of the triangle?
(A) $\pi/2$ (B) $3\pi/4$ (C) $\pi/4$ (D) $\pi/6$
9. The radius of a circle is increasing. At a certain instant the rate of increase of the area is numerically equal to twice the rate of increase of the circumference. What is the radius of the circle at that instant?
(A) 2 (B) 1 (C) 1/2 (D) $\sqrt{2}$

10. The graph of the equation $x^2 + y^2 + 4x - 6y - 3 = 0$ describes a:
 (A) hyperbola with center $(-2,3)$ and vertices $(4,-3)$ and $(-4,3)$
 (B) hyperbola with center $(-2,3)$ and vertices $(2,-3)$ and $(3,-2)$
 (C) circle with center $(-2,3)$ and radius 8
 (D) circle with center $(-2,3)$ and radius 4

11. $\lim_{\theta \rightarrow \pi} \frac{1 + \tan \theta - \cos^2 \theta}{\sin \theta} =$
 (A) 1 (B) -1 (C) ∞ (D) undefined

12. The parametric equation $x(t)$ for the line segment with endpoints $(-2,1)$ and $(3,5)$ where $0 \leq t \leq 1$ is:
 (A) $x = -2 + 5t$ (B) $x = 1 + 4t$ (C) $x = -2 + 4t$ (D) $x = 1 + 5t$

13. If the midpoints of consecutive sides of any quadrilateral are connected the result is always a:
 (A) square (B) trapezoid (C) kite (D) parallelogram

14. An electronics company has 3000 hours to make headphones and media-players. It takes 2 hours to make a pair of headphones, 5 hours to make a media-player and they can at most make 750 headphones or 500 media-players. If they make \$4 profit on headphones and \$7 profit on media-players, what is their maximum possible profit?
 (A) 4708 (B) 4700 (C) 5100 (D) 3500

15. Let f be a function that is everywhere differentiable. The value of $f'(x)$ is given for several values of x in the table at right. If $f'(x)$ is always increasing, which statement about $f(x)$ must be true?

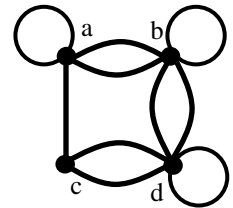
x	-10	-5	0	5	10
$f'(x)$	-2	-1	0	1	2

- (A) $f(x)$ has a relative minimum at $x = 0$. (B) $f(x)$ has a point of inflection at $x = 0$.
 (C) $f(x)$ passes through the origin (D) $f(x)$ is an odd function

16. If $\log_2 x = \log_2 a + \log_2 b - \log_2 c$, then x is:
 (A) 2^{a+b-c} (B) $a + b - c$ (C) $2^a + 2^b - 2^c$ (D) ab/c

17. The position of a particle on the x -axis at time t , $t > 0$, is $\ln t$. The average velocity of the particle for $1 \leq t \leq e$ is:
 (A) 1 (B) $\frac{1}{e} - 1$ (C) $\frac{1}{e - 1}$ (D) e

18. The graph depicted at right is a:
 (A) pseudograph (B) simple graph (C) directed graph (D) directed multigraph



19. The expression $4ab\sqrt{2b} - 3a\sqrt{18b^3} + 7ab\sqrt{6b}$ is equivalent to:
 (A) $2ab\sqrt{6b}$ (B) $-5ab\sqrt{2b} + 7ab\sqrt{6b}$ (C) $-5ab + 7ab\sqrt{6b}$ (D) $16ab\sqrt{2b}$

20. What is the fourth term of the sequence defined by the recursive rule $a_1=3$, $a_n = n + a_{n-1} - 7$?
 (A) -1 (B) -6 (C) -9 (D) -10

21. What type of function is $f(x) = 4e^{0.5x}$?
 (A) power function (B) exponential growth function
 (C) logarithmic function (D) logistic growth function

SCHOLARSHIP TEST 2011 ANSWER KEY

- 1.) D
- 2.) A
- 3.) B
- 4.) C
- 5.) B
- 6.) C
- 7.) D
- 8.) A
- 9.) A
- 10.) D
- 11.) B
- 12.) A
- 13.) D
- 14.) C
- 15.) A
- 16.) D
- 17.) C
- 18.) A
- 19.) B
- 20.) C
- 21.) B