MCTM

# MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
## 2011 MATH CONTEST
# SENIOR

**DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.**

1. If $f(x)=\frac{1}{2}x^2-3$ and $g(x)=2x+5$, what is the value of $(g \circ f)(4)\cdot(f \circ g)(5)$?
   (A) 1642.5 (B) 1956 (C) 2628 (D) 1222.5

2. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t)=3t^3-18t^2+24t$. Over which interval is its average velocity zero?
   (A) between 0 and 2 (B) between 0 and 4
   (C) between 2 and 4 (D) infinite number

3. $\lim_{x\to+\infty}\dfrac{x-\frac{1}{2x}}{2x+\frac{1}{6x}}$
   (A) 1/2 (B) -1/2 (C) -1/3 (D) 2

4. Find real numbers x and y that make $(5-2i)-7=x-(3+yi)$ true.
   (A) x = 1 y = 2i (B) x = 1 y = 2 (C) x = -2 y = -2 (D) x = 1 y = -2

5. What is the sum of the first 50 even integers?
   (A) 1275 (B) 2550 (C) 50 (D) 100

6. Rachel rides her bike through a tunnel shaped like the top half of an ellipse. The tunnel is 11 meters wide and 3 meters high. On her bike, the top of Rachel's helmet is 1.8 meters above the ground. If she were to ride through the tunnel 4.1 meters from the center, would her helmet miss the ceiling? If so, by how much?
   (A) Yes, she would clear the ceiling by 0.2 meters
   (B) Yes, she would clear the ceiling by 0.4 meters
   (C) Yes, she would clear the ceiling by 0.3 meters
   (D) No, she would hit the ceiling

7. Suppose that two airplanes leave from the same airport on the same day using the same flight path. One airplane leaves at 8 a.m. and averages 450 mph and the other leaves at 10 a.m. and averages 540 mph. Assume that both planes maintain a constant speed. What pair of parametric equations best represents this situation?
   (A) x = 450t and x = 540(t - 2) (B) x = 540t and x = 450(t – 2)
   (C) x + y = 450 and x – y = 540 (D) x = 450t and x = 540(t + 2)

8. A pulley of radius 4 inches is rotated through an angle of 700 degrees. How far would a box connected to this pulley rise off the ground?
   (A) 48.87 in (B) 307.05 in (C) 24.43 in (D) 12.92 in

9. Identify the function that is odd.

(A) $g(x) = -2x^4 + 2x^2 - 4$ (B) $p(x) = \dfrac{-2x^3}{-2x^2 + 2}$

(C) $h(x) = |-2x + 2| - 2$ (C) $f(x) = -4x^3 + 4x^2 - 4$

10. $|2 - 3i| =$

(A) $\sqrt{5}$ (B) $\sqrt{13}$ (C) 1 (D) $\sqrt{5i}$

11. What is the horizontal asymptote of the rational function $f(x) = \dfrac{3x^2 - 6x - 8}{2x^2 - x - 1}$?

(A) y=3 (B) y=2 (C) y=1.5 (D) y=8

12. Which equation could determine a curve that, when rotated about an axis, create the 3–dimensional surface used in satellite dishes or the reflective surface of an automobile's headlights?

(A) $4x^2 + 9y^2 = 36$ (B) $4x^2 - 9y^2 = 36$ (C) $9x^2 + 9y^2 = 36$ (D) $4x^2 - 9y = 36$

13. A rubber ball dropped on a hard surface takes a sequence of bounces, each one 4/5 as high as the preceding one. If this ball is dropped from a height of 15 feet, how far will it have travelled when it hits the surface the fifth time?

(A) 50.424 ft (B)100.848 ft (C) 184.152 ft (D) 85.848 ft

14. $\lim\limits_{x \to -1} \dfrac{|x+1|}{x^2 - 1} =$

(A) -1/2 (B) $\infty$ (C) does not exist (D) 1/2

15. Convert the polar equation to rectangular form: $r = 6\cos\theta$

(A) $(x+3)^2 + y^2 = 9$ (B) $y = x$ (C) $(x-3)^2 + y^2 = 9$ (D) $x^2 + (y+3)^2 = 9$

16. Identify the equation of a circle.

(A) $5y^2 - 9y - 12x^2 - 3x + 6 = 0$ (B) $-3x^2 - 9x - 3y^2 + 6y + 1 = 0$

(C) $6y^2 - 9x + y = 8$ (D) $6x^2 - 9x + 12y^2 + y = -1$

17. An unknown parabola intersects the x-axis at -7 and $2\dfrac{1}{2}$. Which equation could not specify the parabola?

(A) $-x^2 - 4\dfrac{1}{2}x + 17\dfrac{1}{2} = y$ (B) $35 - y = 2x^2 + 9x$

(C) $-7x^2 + 2\dfrac{1}{2}x + 4\dfrac{1}{2} = y$ (D) $y = (5x + 35)(2x - 5)$

18. How long is the vector whose tail is at the point (-1, 2, 1) and whose tip is at the point (2, 3, 2)?

(A) $\sqrt{11}$ (B) 11 (C) $\sqrt{5}$ (D) 5

19. What is the simplest form of the expression $(i^{11} + i^{10} + i^9 + i^8)^{16}$?

(A) $16i^{11} + 16i^{10} + 16i^9 + 16i^8$ (B) 1 (C) 0 (D) $2 + 2i$

**SENIOR 2011 Answer Key**

1. A
2. D
3. A
4. B
5. B
6. A
7. A
8. A
9. B
10. B
11. C
12. D
13. D
14. C
15. C
16. B
17. C
18. A
19. C