

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS  
2012 MATH CONTEST

## Scholarship Test

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

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- The graph of the equation  $9x^2 - 4y^2 - 18x - 27 = 0$  describes a  
(A) circle (B) ellipse (C) hyperbola (D) parabola
- What is the product of  $x$ ,  $y$  and  $z$  if  $x + 2y - z = 1$ ,  $2x + z = 6$ , and  $5x - y + 3z = 15$ ?  
(A) 0 (B) 8 (C) 24 (D) 1
- How many different ways can the letters in KALISPELL be arranged?  
(A) 120960 (B) 362880 (C) 181440 (D) 60480
- There are 7 people at a gathering. Each person shakes hands with every other person. How many handshakes are required?  
(A) 7 (B) 14 (C) 21 (D) 42
- Evaluate the series  $20 - 5 + 1.25 - 0.1325 + \dots$   
(A) 17 (B) 16 (C) 15 (D) 14
- Determine  $\lim_{x \rightarrow 2} f(x)$ , where
$$f(x) = \begin{cases} 2x + 21 & \text{if } x < 2 \\ 10 & \text{if } x = 2 \\ x^2 + 6x + 9 & \text{if } x > 2 \end{cases}$$
  
(A) 2 (B) 10 (C) 25 (D) does not exist
- If  $x = \frac{1}{3}t$  and  $y = 5t + 2$  determine the function  $y(x)$ .  
(A)  $y = \frac{5}{3}x + 2$  (B)  $y = \frac{5}{3}x + \frac{2}{3}$  (C)  $y = \frac{5}{3}x^2 + \frac{5}{3}x$  (D)  $y = 15x + 2$
- Give the maximum possible sum of two negative numbers whose product is 144.  
(A) -24 (B) -20 (C) -40 (D) -32
- Evaluate the derivative of  $y = e^\pi + e^x$  at  $x = \ln 5$ .  
(A)  $\pi e^{\pi-1} + 5$  (B) 5 (C) 6 (D)  $5e^\pi$
- Find the integral  $\int 3x^5 + \sin x \, dx$   
(A)  $0.5x^6 - \cos x + C$  (B)  $15x^4 + \cos x + C$  (C)  $0.6x^6 - \cos x + C$  (D)  $15x^6 + \cos x + C$
- If  $f(3) = 2$ ,  $f'(3) = 4$ ,  $g(3) = 1$ ,  $g'(3) = 3$ , and  $h(x) = f(x)g(x)$ , then what is  $h'(3)$ ?  
(A) 14 (B) 12 (C) 10 (D) 2

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12. In a 200-meter race there are 8 competitors. How many different ways can they finish in 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> places?  
 (A) 56 (B) 40320 (C) 336 (D) not enough information

13. Determine the foci and focal constant of the graph of  $9x^2 + 4y^2 - 18x - 27 = 0$   
 (A)  $(-\sqrt{5}, 1), (\sqrt{5}, 1), 6$  (B)  $(1, -\sqrt{5}), (1, \sqrt{5}), 4$   
 (C)  $(1, \sqrt{5}), (1, -\sqrt{5}), 6$  (D)  $(-\sqrt{5}, 1), (\sqrt{5}, 1), 4$

14. Evaluate the following limit.

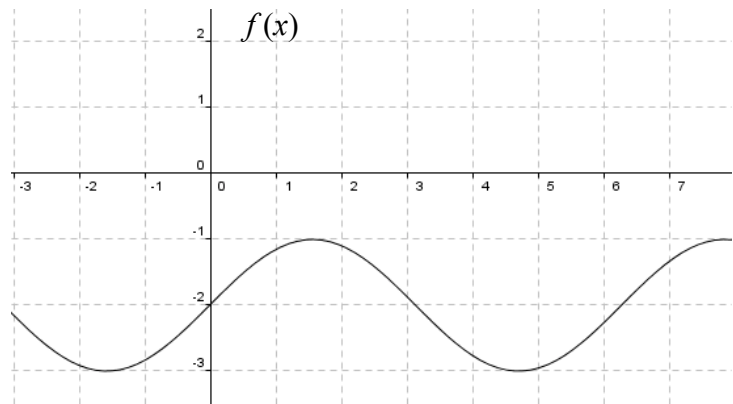
$$\lim_{x \rightarrow -4} \frac{(x-3)(x-1)(x+4)}{x^2+x-12}$$

(A) -5 (B) 0 (C) 4 (D) undefined

15. Use the graph at the right to estimate the following integral.

$$\int_{-2}^4 f(x) dx$$

- (A) 14 (B) 10  
 (C) -7 (D) -12



16. Write the series  $3 - 8 + 15 - 24 + \dots$  in sigma-notation form.

(A)  $\sum_{n=2}^{\infty} (-1)^{n+1} (n^2 - 1)$  (B)  $\sum_{n=1}^{\infty} (-1)^n (n^2 + 2)$  (C)  $\sum_{n=0}^{\infty} (-1)^n (n^2 + 3)$  (D)  $\sum_{n=2}^{\infty} (-1)^n (n^2 - 1)$

17. Consider the region bounded by the curve  $y = \sqrt{x}$  and the  $x$ -axis. Find the area of the largest rectangle possible with one corner at  $(2, 0)$  and its opposite corner on the curve to the left of  $x = 2$ .  
 (A) 1 (B) 1.09 (C) 1.59 (D) 1.89

18. If  $f(1) = 5$  and  $f'(2) = -2$ , find the equation of the tangent line to  $f(x)$  at  $x = 2$ .  
 (A)  $y = -2x + 7$  (B)  $y = 2x + 3$  (C)  $y = -2x + 11$  (D)  $y = 2x - 9$

19. Find  $\lim_{h \rightarrow 0} \frac{(x+1+h)^{\frac{2}{3}} - (x+1)^{\frac{2}{3}}}{h}$   
 (A) 0 (B)  $\frac{2}{3\sqrt[3]{x+1}}$  (C)  $\frac{2}{3}(x+1)^{\frac{4}{3}}$  (D) undefined

20. Find  $\int_{-2}^5 dx$   
 (A) 3 (B) -10 (C) 0 (D) 7

## Scholarship Test 2012 Answer Key

1. C
2. B
3. D
4. C
5. B
6. C
7. D
8. A
9. B
10. A
11. C
12. C
13. C
14. A
15. D
16. D
17. B
18. A
19. B
20. D