



MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2014 MATH CONTEST
TEAM 11-12

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

1. A calculus teacher calculates the mean of the latest test scores to be 68 and the standard deviation to be 10. Realizing that the test scores are below average for his class, the teacher decides to raise everyone's score by 8 points. What is the mean and standard deviation of the higher test scores?
(A) mean: 68, standard deviation: 10 (B) mean: 68, standard deviation: 18
(C) mean: 76, standard deviation: 10 (D) mean: 76, standard deviation: 18
(E) none of these

2. You join a health club which charges you a \$225 registration fee plus \$30 per month charge. Which is the first month you average less than \$50 per month for membership?
(A) month 11 (B) month 12 (C) month 13
(D) month 14 (E) none of these

3. Which equation has the largest number of solutions over the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$?
(A) $\sin(x) = \frac{\sqrt{2}}{2}$ (B) $\cos(x) = -\frac{\sqrt{3}}{2}$ (C) $\sin(x) = \cos(3x) + 1$ (D) $\sin(x) = -\frac{1}{2}$
(E) all have the same number of solutions

4. The complement of an angle is 3 times the angle. What is the angle in degrees?
(A) 45° (B) 60° (C) 30° (D) 22.5° (E) none of these

5. Evaluate $\csc\left(-\frac{10\pi}{3}\right)$
(A) $\frac{2\sqrt{3}}{3}$ (B) 2 (C) -2 (D) $-\sqrt{2}$ (E) none of these

6. A sophomore class of 380 students took a science test. The test scores followed an approximately normal distribution. Their mean score was 82% with a standard deviation of 4%. About what percent of students scored between 78% and 90%?
(A) 99.5% (B) 95% (C) 81.5% (D) 68% (E) none of these

7. Evaluate the limit, $\lim_{x \rightarrow \infty} \left(\frac{e^x + x^2}{5^x} + \frac{3x}{x+1} \right)$
(A) 0 (B) 3 (C) 4 (D) ∞ (E) none of these

8. A ball is dropped from a height of 8 feet. Each time the ball hits the ground it bounces to 75% of its previous height. Find the total distance traveled by the ball.
(A) 48 ft (B) 72 ft (C) 54 ft (D) 56 ft (E) none of these

9. For the opening day of a movie, a local theater sold 756 tickets. The receipts totaled \$5409. Tickets for children cost \$6.50 each, tickets for adults are \$8.50 each, and tickets for senior citizens are \$7.00 each. There were twice as many adult tickets sold as senior citizen tickets. What is the sum of the adult and senior citizens who attended the movie on opening day?
(A) 111 (B) 330 (C) 426 (D) 300 (E) none of these

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10. Solve for x and round the solution to the hundredth place: $\log_5(3x) + \log_5(x - 1) = 3$
(A) 6.97 (B) 18.76 (C) 3.14 (D) 31.00 (E) none of these
11. Which equation has no solution?
(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$
(E) none of these
12. Determine the value of x in the following equation: $3 \cdot 4^{x/3} - 20 = 172$
(A) 48 (B) 6 (C) 3 (D) 9 (E) none of these
13. Suppose that y is directly proportional to x^2 and z is inversely proportional to y . When $x = 2$, $y = 12$ and $z = 3$. Determine the value of z when $x = 3$.
(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) 4 (D) 3 (E) none of these
14. Identify the foci of the conic section that is defined by the equation: $9x^2 + 4y^2 - 144 = 0$
(A) $(\pm 2\sqrt{5}, 0)$ (B) $(0, \pm 2\sqrt{5})$ (C) $(0, \pm 2\sqrt{13})$
(D) $(0, \pm 2\sqrt{3})$ (E) none of these
15. Solve for x in the inequality: $\frac{1}{4}\left(3x + \frac{3}{5}\right) > -\frac{4}{5}x + \frac{1}{2}$
(A) $\left(-\infty, \frac{7}{31}\right)$ (B) $\left(\frac{7}{31}, \infty\right)$ (C) $\left(-\infty, \frac{-7}{31}\right)$
(D) $\left(\frac{24}{31}, \infty\right)$ (E) none of these
16. Determine the probability that on a 5 question true/false exam you get exactly 4 problems correct by randomly guessing.
(A) $4/5$ (B) $1/32$ (C) $27/32$ (D) $5/32$ (E) none of these
17. Find the next number in the geometric sequence of complex numbers:
 $3 - 2i, -2 - 3i, -3 + 2i, 2 + 3i, \dots$
(A) $3 - 2i$ (B) $-2 - 3i$ (C) $-3 + 2i$ (D) $2 + 3i$ (E) none of these
18. The half-life of carbon-14 is about 5700 years. Find the age, to the nearest year, of a sample of carbon-14 where 38% of the carbon-14 originally present has decayed.
(A) 3931 yrs (B) 7957 yrs (C) 2166 yrs (D) 7166 yrs (E) none of these
19. A right circular cylinder has a height that is twice the radius of its base. What fraction of the cylinder is contained in a sphere that has a radius equal to the radius of the base of the cylinder?
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$ (E) none of these

TEAM 11-12 2014 Answer Key

1. C
2. B
3. C
4. D
5. A
6. C
7. B
8. D
9. B
10. A
11. D
12. D
13. A
14. B
15. B
16. D
17. A
18. A
19. D