

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2014 MATH CONTEST
SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

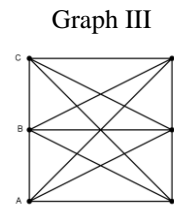
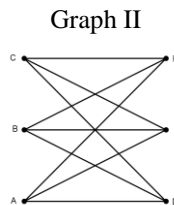
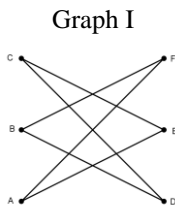
- If $h(x) = -4 \cos(2x + \pi) - 4$, which of the following is false?
(A) $h'(x) = 8\sin(2x + \pi)$ (B) amplitude = -4 (C) period = π
(D) phase shift = $-\frac{\pi}{2}$ (E) none of these
- Identify the conic section represented by $4x^2 - 2xy - 5y^2 + 3x - 6y = 18$.
(A) parabola (B) ellipse (C) circle (D) hyperbola (E) none of these
- $x^2 + y^2 = 25$ is translated as follows: $(x, y) \rightarrow (x - 2, y + 4)$. Which equation represents the translated conic?
(A) $x^2 + y^2 + 4x - 8y - 5 = 0$ (B) $x^2 + y^2 - 8x + 4y + 5 = 0$
(C) $x^2 + y^2 - 4x + 8y - 5 = 0$ (D) $x^2 + y^2 - 9 = 0$
(E) none of these
- Given the parametric equations $x = 4 \sin t$ and $y = 5 \cos t$ ($0 \leq t \leq 2\pi$), y is graphed as a function of x . What is the shape of the resulting graph?
(A) Ellipse (B) Hyperbola (C) Parabola (D) Sinusoidal (E) none of these
- A football is kicked from the ground with an initial speed of 85 ft/second at an angle of 35° . Assuming a level field and no air resistance, how far (to the nearest foot) will the football travel horizontally before hitting the ground?
(A) 212 ft (B) 179 ft (C) 156 ft (D) 69 ft (E) none of these
- Determine the value of the limit: $\lim_{x \rightarrow \infty} \frac{\sin(3x)}{x}$
(A) 3 (B) 1 (C) 0 (D) does not exist (E) none of these
- If $f(x) = \begin{cases} ax + 2 & x > 3 \\ bx^2 + 1 & x < 3 \end{cases}$, find the value of the one one-sided limit: $\lim_{x \rightarrow 3^+} f(x)$
(A) $3a + 2$ (B) $3b + 1$ (C) $9b + 1$ (D) does not exist (E) none of these
- The point(s) on the curve $x^2 - y^2 = 9$ closest to the point $(7, 0)$ is (are)
(A) $(4, \sqrt{7})$ (B) $(\frac{9}{2}, \frac{3\sqrt{5}}{2})$ (C) $(\pm 3, 0)$ (D) $(\frac{7}{2}, \frac{\pm\sqrt{13}}{2})$ (E) none of these
- A furniture manufacturer makes both stools and rocking chairs. The company needs to produce at least 30 rocking chairs and 40 stools a day. They must also meet their demand for at least twice as many stools as rocking chairs. They can produce no more than 120 pieces of furniture each day. If each stool sells for \$80 and each rocking chair sells for \$120, what is the maximum daily income?
(A) \$14,400 (B) \$11,200 (C) \$12,800 (D) \$7,200 (E) none of these
- Given that the graph of function f passes through $(2, 28)$ and that the slope of its tangent line at $(x, f(x))$ is $4x + 1$, find $f(1)$.
(A) 4 (B) 5 (C) 13 (D) 21 (E) none of these
- What is the area of the region bound by the graphs of $y = 2x^2$ and $y = 4x$?
(A) 8 (B) $\frac{8}{3}$ (C) $\frac{10}{3}$ (D) π (E) none of these

SCHOLARSHIP TEST 2014 page 2

12. On the interval $[1, 3]$, identify the x -value where the instantaneous rate of change for the function $f(x) = 2x^3 - x$ is equal to its average rate of change.
 (A) $\frac{11}{6}$ (B) 2 (C) $\frac{5\sqrt{6}}{6}$ (D) $\frac{\sqrt{39}}{3}$ (E) none of these
13. If $y = \sin^3 x$, then $dy/dx =$
 (A) $3 \sin^2 x$ (B) $-3 \sin^2 x$ (C) $3 \cos^2 x$ (D) $-3 \cos^2 x$ (E) none of these
14. Find the equation of the line tangent to the function $y = \frac{4x+6}{2x}$ at the point $(1, 5)$.
 (A) $3x + y = 8$ (B) $3x + y = 2$ (C) $2x - y = -3$ (D) $x + 3y = 16$ (E) none of these
15. If nine wood tiles stamped with the letters K, A, L, I, S, P, E, L, L are randomly drawn from a container and arranged left to right as they are drawn, what is the probability that they will spell the word KALISPELL?
 (A) $\frac{1}{362,880}$ (B) $\frac{1}{181,440}$ (C) $\frac{1}{120,960}$ (D) $\frac{1}{60,480}$ (E) none of these
16. Throughout the season, Jill makes 70% of her free throw attempts on average. What is the probability that she will make two of the next three attempts?
 (A) 0.784 (B) 0.559 (C) 0.441 (D) 0.189 (E) none of these
17. If $a_{n+1} = -\frac{1}{2}a_n$ and $a_1 = -100$, find:

$$\sum_{n=1}^5 a_n$$

 (A) -6.25 (B) -68.75 (C) -67.5 (D) $-66\frac{2}{3}$ (E) none of these
18. A tree is planted with an original height of 3 feet. The first year it grows $\frac{1}{2}$ of a foot. Each subsequent year, it grows 97% as much as the year before. How tall can the tree grow?
 (A) $19\frac{2}{3} ft$ (B) $16\frac{2}{3} ft$ (C) 45.6 ft (D) 5.2 ft (E) none of these
19. Which of the following graphs can be drawn without lifting your pencil or repeating a line?



- (A) Just I and III (B) Just I (C) Just II and III (D) Just III (E) All of them (F) None of them

SCHOLARSHIP TEST 2014 ANSWER KEY

1. B
2. D
3. C
4. A
5. A
6. C
7. A
8. D
9. B
10. D
11. B
12. D
13. E
14. A
15. D
16. C
17. B
18. A
19. A