

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS  
2014 MATH CONTEST  
SENIOR

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

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- Determine which of the following is equivalent to  $(i^6 + i^5 + i^4 + i^3 + i^2 + i)^2$ .  
(A) 1      (B) -1      (C)  $-2i$       (D)  $2i$       (E) none of these
- Which two complex numbers have a sum of 6 and a product of 34?  
(A)  $(4+i)$  and  $(4-i)$       (B)  $(3+i)$  and  $(3-i)$       (C)  $(6+i)$  and  $(6-i)$   
(D)  $(3+5i)$  and  $(3-5i)$       (E) none of these
- Find the value of  $b$  so that the complex number  $3+bi$  is 4 units from the origin in the complex plane.  
(A) 1      (B) 7      (C)  $\sqrt{5}$       (D)  $\sqrt{7}$       (E) none of these
- What is the equation of the line of symmetry of the parabola  $y = \frac{1}{2}x^2 - 4x + 13$ .  
(A)  $x = 4$       (B)  $y = 5$       (C)  $x = -4$       (D)  $x = 5$       (E) none of these
- Determine the shape of the graph of the equation  $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{49} = 1$   
(A) circle      (B) hyperbola      (C) parabola      (D) ellipse      (E) none of these
- Write the equation of the ellipse  $9x^2 + 4y^2 - 36x + 24y + 36 = 0$  in standard form.  
(A)  $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$       (B)  $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$       (C)  $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$   
(D)  $(x-2)^2 + (y+3)^2 = 36$       (E) none of these
- Which of the following functions have an absolute maximum?  
 $f(x) = 2x^2 - 3x + 9$        $g(x) = -3x^3 + 9x - 11$        $h(x) = -12x^4 - 3x^3 - 5x^2$        $j(x) = -3x^5 - 2x^4 - 3x^3$   
(A)  $f(x)$ ,  $g(x)$ ,  $h(x)$  and  $j(x)$       (B)  $f(x)$  and  $h(x)$  only      (C)  $g(x)$  and  $j(x)$   
(D)  $h(x)$  only      (E) none of these
- What is the minimal degree of a polynomial function with roots  $\frac{1}{2}$ ,  $-\frac{3}{5}$ ,  $\sqrt{3}$  and  $(2-5i)$ ?  
(A) 4      (B) 5      (C) 6      (D) 7      (E) none of these
- $\overline{AB}$  is a vector from  $A(3,-1)$  to  $B(5,-4)$ . What is the magnitude of  $\overline{AB}$  to the nearest hundredth?  
(A) 5      (B) 13      (C) 3.61      (D) 2.34      (E) none of these
- A 24 N force acting at  $30^\circ$  E of N and a 32 N force acting at  $45^\circ$  N of E act concurrently on a point. What is the magnitude and direction of a third force that most closely produces equilibrium at the point?  
(A) 56 N;  $30^\circ$  E of N      (B) 56 N;  $37.5^\circ$  W of S      (C) 55.5 N;  $37.1^\circ$  W of S  
(D) 55.5 N;  $38.6^\circ$  W of S      (E) 56 N;  $37.1^\circ$  N of E

11. Determine which represents the equation of the line described by the parametric equations  $x = 8t + 3$   
 $y = -3t + 4$
- (A)  $y = \frac{3}{8}x + \frac{27}{8}$                       (B)  $y = 5x + 7$                       (C)  $y = -\frac{8}{3}x + \frac{3}{4}$   
 (D)  $y = -\frac{3}{8}x + \frac{3}{4}$                       (E) none of these

12. A shot put leaves the thrower's hand at a height of 5 feet and a velocity of 35 feet per second at an angle of 40 degrees above the horizontal. Which of the following could be used to model the path of the shot put  $t$  seconds after leaving the thrower's hand?

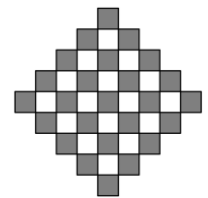
- (A)  $x = 35t \cos(50)$   
 $y = 35t \sin(50) - 16t^2 + 5$                       (B)  $x = 35t \cos(40)$   
 $y = 35t \sin(40) - 16t^2 + 5$                       (C)  $x = 35t \sin(-40)$   
 $y = 35t \cos(-40) - 16t^2$   
 (D)  $x = 35t \sin(40)$   
 $y = 35t \cos(40) - 16t^2 + 5$                       (E) none of these

13. Evaluate  $\lim_{x \rightarrow 5} \frac{2x^2 - 9x - 5}{x - 5}$
- (A) 11                      (B) 0                      (C) 1                      (D)  $\infty$                       (E) none of these

14. Evaluate  $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x + 5}{2x^3 + 4x^2 + 2}$
- (A) 5                      (B) 0                      (C)  $\frac{3}{2}$                       (D)  $\infty$                       (E) none of these

15. You are designing a closed cardboard box with a top, a bottom and four sides. The length of the box must be twice its width and the volume of the box must be 576 cubic inches. What is the surface area of the box that uses the least amount of cardboard?
- (A)  $864 \text{ in}^2$                       (B)  $216 \text{ in}^2$                       (C)  $6 \cdot 288^{2/3} \text{ in}^2$                       (D)  $432 \text{ in}^2$                       (E) none of these

16. The pattern at right is made up of black and white tiles. It is 9 tiles across. In the town hall there is a pattern like it that is 45 tiles across. How many tiles will it contain altogether?
- (A) 4052                      (B) 1013                      (C) 2026  
 (D) 2025                      (E) none of these



17. Find the 2014<sup>th</sup> term in the following sequence: 3,  $\frac{11}{2}$ , 8,  $\frac{21}{2}$ , 13, ...
- (A) 10071                      (B) 5035                      (C)  $\frac{10071}{2}$                       (D)  $\frac{10073}{2}$                       (E) none of these

SENIOR 2014 Answer Key

1. C
2. D
3. D
4. A
5. B
6. A
7. D
8. C
9. C
10. D
11. E
12. B
13. A
14. B
15. D
16. B
17. C