

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2015 MATH CONTEST
SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

- Find the slope of the tangent line to the curve $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at the point (3,0).
A) $\frac{16}{9}$ B) 1 C) 0 D) 16 E) none of these
- A fish pond has an initial population of 500 fish. 25% of the fish are harvested or die each year and 60 fish are planted. The number of fish will stabilize to approximately what number over a long period of time?
A) 60 B) 240 C) 380 D) 560 E) none of these
- A person on a rope swing travels 20 feet (total distance) on the first swing. On every successive swing the person only travels 85% of the previous swing. If they swing until they come to a stop how far will they have travelled?
A) The distance will become infinite. B) 80ft 6in C) 120ft D) 133ft 4in
E) none of these
- Evaluate $\sum_{n=5}^9 2n - 3$
A) 55 B) 11 C) 15 D) 22 E) none of these
- Identify the conic section represented by $4x^2 - 2xy + 5y^2 + 3x - 6y = 18$.
A) Parabola B) ellipse C) circle D) hyperbola E) none of these
- There is a 10% chance a person is left handed and a 90% chance they are right handed. If five people are picked at random what is the probability to the nearest percent that exactly two would be left handed?
A) 1% B) 5% C) 7% D) 10% E) none of these
- $y^2 - x = 3^2$ is transformed as follows: $(x, y) \rightarrow (-2x, y - 3)$. Which equation represents the transformed conic?
A) $y^2 = -2x$ B) $y = 2x$ C) $y^2 - 6y + 2x = 0$ D) $y^2 - 3y - 2x = 9$
E) none of these
- A piece of string is cut and the two resulting pieces are shaped into a square and a circle. If the string is 60 inches long, how should it be cut, to the nearest 0.1 in, to minimize the total area? The amount used for the circle should be:
A) 40 in B) 30 in C) 26.4 in D) 18.7 in E) none of these
- How many different committees of four students can be formed in a class consisting of 24 total students?
A) $6.2 \cdot 10^{23}$ B) 255,024 C) 10,626 D) 96 E) none of these
- How many unique six letter arrangements can be formed from the letters in SISTER that end with the letter T?
A) 720 B) 360 C) 120 D) 60 E) none of these

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11. Marci and Heather are both enrolled in the same class. The class consists of twelve students and is randomly divided into three equal groups. To the nearest percent, what is the probability that Marci and Heather are in the same group?

- A) 17% B) 13% C) 9% D) 1% E) none of these

12. Consider the function: $f(x) = \begin{cases} 4 & \text{if } x > 2 \\ 8 & \text{if } x = 2. \\ 3x & \text{if } x < 2 \end{cases}$ Which of the following is true?

- A) $\lim_{x \rightarrow 2^-} f(x) = 4$ B) $\lim_{x \rightarrow 2^-} f(x) = 6$ C) $\lim_{x \rightarrow 2^-} f(x) = 8$
 D) $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$ E) none of these

13. Evaluate the limit: $\lim_{x \rightarrow a} \frac{x-a}{a}$ where $a \neq 0$.

- A) 0 B) a C) $+\infty$ D) $-\infty$ E) none of these

14. For what value of the constant c is the function f(x) continuous?

$$f(x) = \begin{cases} cx + 6 & \text{if } x \leq 4 \\ cx^2 - 6 & \text{if } x > 4 \end{cases}$$

- A) $c = 1$ B) $c = 2$ C) $c = 10$ D) $c = 0.6$ E) none of these

15. If $y = \frac{x^2+2}{x-1}$, then $\frac{dy}{dx} =$

- A) $2x$ B) $x - 2$ C) $\frac{x^2-2x+2}{x^2-2x+1}$ D) $\frac{x^2-2x-2}{(x-1)^2}$ E) none of these

16. Susan runs at an average speed of 7 miles per hour. Two hours after Susan leaves your house, you leave in your car and follow the same route. If your average speed is 35 miles per hour, how far from your house will you be when you catch up to Susan? Round to the nearest mile

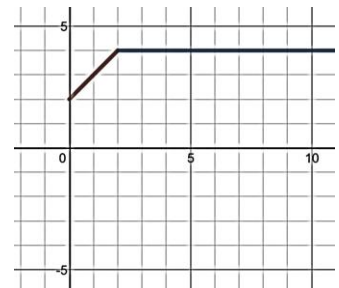
- A) 3 miles B) 5 miles C) 14 miles D) 18 miles E) none of these

17. If f(t) is measured in liters/second and t is measured in seconds, what are the units of $\int_2^4 f(t)dt$?

- A) liters/second B) liters C) seconds
 D) liters/second/second E) none of these

18. Use the graph of function f shown to the right to evaluate $\int_1^6 f(x)dx$.

- A) 15.5 B) 17 C) 19.5 D) 22
 E) none of these



19. Which is equivalent to the integral $\int_1^4 2f'(a) da$?

- A) $2f'(3)$ B) $2f(3)$ C) $2(f'(4) - f'(1))$
 D) $2f(4) - 2f(1)$ E) none of these

SCHOLARSHIP TEST 2015 ANSWER KEY

1. E
2. B
3. D
4. A
5. B
6. C
7. C
8. C
9. C
10. D
11. C
12. B
13. A
14. A
15. D
16. D
17. B
18. C
19. D