

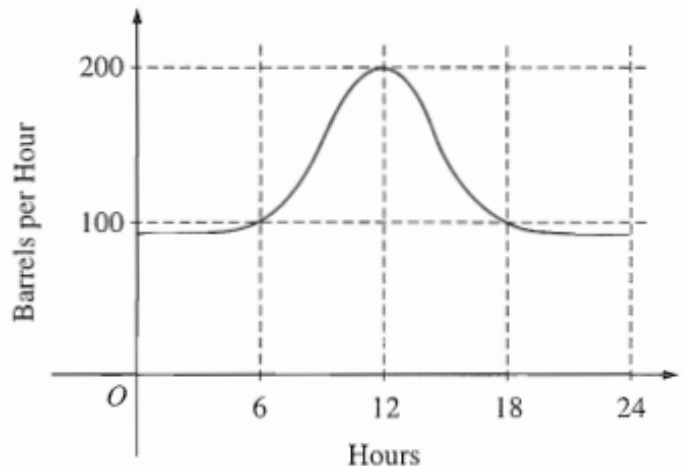
MONTANA COUNCIL OF TEACHERS OF MATHEMATICS  
2016 MATH CONTEST  
TEAM 11-12

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

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- The complement of an angle is 5 times the angle. What is the angle in degrees?  
A) 18                      B) 15                      C) 22.5                      D) 30                      E) None of these
- A square is inscribed in a circle. Find the perimeter of the square in terms of the radius of the circle.  
A)  $8r$                       B)  $4r\sqrt{2}$                       C)  $4\pi r$                       D)  $4\pi r^2$                       E) None of these
- While training for a biathlon race, Kevin covered a total distance of 9 km by swimming for 45 minutes and running for 20 minutes. The next day he swam for 30 minutes and ran for 40 minutes, covering 14 km. Find his rates (in km/hr) for swimming and running (Assume that these rates are constant.)  
A) Swim : 4                      B) Swim: 10; C) Swim:3;                      D) Swim: 4;                      E) None of these  
Run: 18                      Run: 14                      Run: 18                      Run: 14
- How many three-digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if no digit may be repeated?  
A) 720                      B) 504                      C) 648                      D) 125                      E) None of these
- A boat sails from point A due west for eight km. The boat then changes course and sails due north for 6 km. If the boat begins to sink, how fast would a rescue ship have to travel from A to reach the ship in 40 minutes?  
A) 5km/hr                      B) 10 km/hr                      C) 15 km/hr                      D) 20km/hr                      E) None of these
- If  $\sqrt[3]{\sqrt{a^8}} = 16$ , then  $a =$   
A) 2                      B) 4                      C) 8                      D) 16                      E) None of these
- Find the equation of the line tangent to the curve  $y = x^2 - 3x + 5$  at the point (1,5).  
A)  $y = -x + 6$                       B)  $y = x + 4$                       C)  $y = 2x + 3$                       D)  $y = -3x + 8$                       E) None of these
- The vertices of quadrilateral ABCD are at the points A(-2,-5), B(6,-5), C(6,1), D(-2, 1). What are the coordinates of the point of intersection of the diagonals?  
A) (2,-3)                      B) (-2, 5)                      C) (0, 0)                      D) (2,-2)                      E) None of these
- The World Series of baseball, a famous sporting event, is played between two teams. As soon as either team wins 4 games, that team is declared World Champions (no game can end in a tie). If a World Series is played between two teams of equal ability (so that each team's probability of winning any game is  $\frac{1}{2}$ ), what is the probability that World Champions are declared after only 4 games.  
A)  $\frac{1}{4}$                       B)  $\frac{3}{8}$                       C)  $\frac{1}{32}$                       D)  $\frac{1}{8}$                       E) None of these
- What is the smallest positive number  $x$  for which  $\tan^2 x + \sec^2 x = 1$ . (Answer is expressed in radians.)  
A)  $2\pi$                       B)  $\frac{\pi}{2}$                       C)  $\pi$                       D)  $\frac{\pi}{4}$                       E) None of these
- If  $x$  is not the square of an integer, then the value of  $7 + \sqrt{x} + \frac{1}{5-\sqrt{x}}$  is a rational number for only one positive integer  $x$ . Find the integer.  
A) 13                      B) 26                      C) 0                      D) 25                      E) None of these

12. What is the sum of all integers  $x$  for which  $\frac{3x+25}{2x-5}$  is an integer?  
 A) 40                      B) 13                      C) 20                      D) 65                      E) None of these
13. Two resistors have impedance  $4 + i$  ohms and  $3 - 2i$  ohms. The total impedance of the two resistors is given by the quotient of their product and their sum. Find the total impedance (in ohms) of these two resistors.  
 A)  $\frac{25}{16} - \frac{5}{16}i$                       B)  $\frac{103}{50} - \frac{21}{50}i$                       C)  $6 - 4i$                       D)  $\frac{103}{221} - \frac{21}{221}i$                       E) None of these
14. Mario wants to build a garden, but he has rabbits that might eat it. He decides to enclose the garden with a fence. The rectangular garden is next to the house so Mario only needs to fence three sides. Fencing is \$15 per foot and he has \$900 to spend. What is the maximum area of the garden given these restrictions?  
 A) 400 ft.<sup>2</sup>                      B) 450 ft.<sup>2</sup>                      C) 500 ft.<sup>2</sup>                      D) 390 ft.<sup>2</sup>                      E) None of these
15. In a given triangle  $\angle A = 60^\circ$  and  $\angle B = 70^\circ$ . The side opposite  $\angle A$ ,  $a$ , is length 4cm. Find the perimeter of the triangle to the nearest tenth of a unit.  
 A) 4.2 cm                      B) 11.9 cm                      C) 15.6 cm                      D) 15.9 cm                      E) None of these
16. Simplify the expression:  $\frac{x^{\frac{7}{4}}y^{-\frac{1}{2}}}{z^{\frac{1}{3}}} * \frac{x^{-\frac{3}{4}}y^{-\frac{1}{2}}z^{\frac{5}{3}}}{x^0}$   
 A)  $\frac{xz^{\frac{3}{4}}\sqrt{z}}{y}$                       B)  $\sqrt[4]{yz^{\frac{5}{9}}}$                       C)  $-2xyz^2$                       D)  $\frac{x^{\frac{1}{4}}z^{\frac{4}{3}}}{y}$                       E) None of these
17. A factory employs over a thousand workers, of whom 35% are female. If 12 members are chosen for a quality control committee from the workers at random, the number of females on the committee would be approximately distributed by a binomial distribution with  $n = 12$  and  $p = 0.35$ . Under this assumption, what is the probability that at most two members of the committee are female?  
 A) 0.1088                      B) 0.0424                      C) 0.7922                      D) 0.8912                      E) None of these
18. Alex wants to solve the equation  $2x^2 + 7x + 5 = 0$  by completing the square. Give the value of  $c$  in this version of the equation.  $(x + c)^2 = \frac{9}{16}$ .  
 A)  $\frac{7}{2}$                       B)  $\frac{49}{16}$                       C)  $\frac{49}{4}$                       D)  $\frac{7}{4}$                       E) None of these
19. The flow of oil, in barrels per hour, through a pipeline on April 1 is given by the graph at right. Which best approximates the number of barrels of oil that passed through the pipeline that day?  
 A) 500                      B) 600  
 C) 2400                      D) 3000  
 E) none of these



## **TEAM 11-12 2016 Answer Key**

1. B
2. B
3. A
4. B
5. C
6. C
7. A
8. D
9. D
10. C
11. B
12. C
13. B
14. B
15. B
16. A
17. E
18. D
19. D