MONTANA COUNCIL OF TEACHERS OF MATHEMATICS

2017 MATH CONTEST

SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

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1. Estimate the area under the function $f(x) = 4x^3$ on the interval $0 \le x \le 2$. Find this estimate using four rectangles of equal width, where the height of each is determined by the midpoint of the subinterval.

A) 14.75 **B)** 15.5 **C)** 16 **D)** 22.75 **E)** none of these

2. Determine the <u>fourth term</u> in the expansion: $\sum_{n=0}^{9} \frac{9!}{n!(9-n)!}$

A) 0.5 **B)** 84 **C)** 126 **D)** 512 **E)** none of these

3. Written as a decimal rounded to the nearest whole number, what is the 18th term of the sequence? $\frac{1}{2}, \frac{3}{5}, \frac{9}{8}, \frac{27}{11}, \ldots$

A) 329 **B)** 675,184 **C)** 2,436,607 **D)** 6,918,223 **E)** none of these

4. Which of the following infinite series will not converge to a finite sum?

A) $0.8 + 0.7 + 0.6 + 0.5 + \cdots$ **B)** 8 + (-4) + 2 + (-1) + … **C)** 1024 + 768 + 576 + 432 + …

D) $\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \cdots$ **E)** none of these

5. What is the maximum value of the function f(x,y) = 3y – 5x under the following constraints?

$$y \ge 4, \quad x \le 12, \quad y \le x + 6, \quad 5x + 3y \ge 54$$

A) -30 **B)** -6 **C)** 9 **D)** 48 **E)** none of these

6. Find the coordinates for the center of the ellipse: $2x^2 - 12x + y^2 + 6y = 10$

A) (-6, 3) **B)** (6, -3) **C)** (-3, 3) **D)** (3, -3) **E)** none of these

7. For what value of k does the following system have exactly three solutions? $\begin{cases} \frac{(x-2)^2}{1} + \frac{y^2}{16} = 1 \\ x = 4y^2 + k \end{cases}$

A) k = -3 **B)** k = -1 **C)** k = 1 **D)** k = 4 **E)** none of these

8. Find y' for the equation $x^2 + y^2 + 2y = 16$.

A) $y' = \frac{-x}{1+y}$ **B)** $y' = -x - y$ **C)** $y' = \frac{8-x}{1+y}$ **D)** $y' = \frac{7-x}{y}$ **E)** none of these

9. What is the slope of the tangent line to the function $y = 2x^3 + \sqrt{1-x} + 10$ at $x = 0$?

A) -0.5 **B)** 0 **C)** 0.5 **D)** 10.5 **E)** none of these

10. Find the x-values of all the points where the graph of $f(x) = 2x^3 + 3x^2 - 5$ has a slope of 12.

A) {-1,0} **B)** {-3,2} **C)** {-2,1} **D)** {-2,2} **E)** none of these

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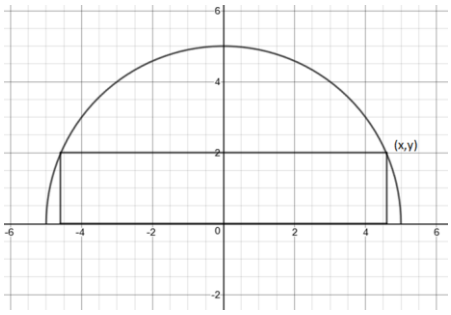
11. A rectangle is positioned with two points on the semicircle $y = \sqrt{25 - x^2}$ as shown. Find the point (x,y) so that the area of the rectangle is a maximum. Round answers to the hundredths place.



A) (3.54, 3.54) **B)** (2.24, 4.47) **C)** (4.47, 2.24)
D) (2.89, 4.08) **E)** none of these

12. How many different committees of 5 people are possible from a group of 12 board members?

A) 120 **B)** 792 **C)** 5040 **D)** 95040 **E)** none of these

13. Cece's Ice Cream shop advertises 116,280 ways of building an ice cream cone with 4 flavours in any order. Determine how many ice cream flavours are available at Cece's Ice Cream shop.

A) 18 **B)** 22 **C)** 26 **D)** 30 **E)** none of these

14. Evaluate $\lim_{x \to 3^+} f(x)$ where $f(x) = \begin{cases} x^2 - 2, & 0 < x < 3 \\ 2, & x = 3 \\ 2x + 3, & 3 < x \end{cases}$.

A) 0 **B)** 2 **C)** 7 **D)** 9 **E)** none of these

15. Determine $\lim_{x \to -5} \frac{x^2+3x-10}{x+5}$.

A) –7 **B)** 0 **C)** 1 **D)** 5 **E)** none of these

16. Determine $\lim_{a \to 0} \frac{\cos(x+a)-\cos x}{a}$

A) $\cos x$ **B)** $-\cos x$ **C)** $\sin x$ **D)** $-\sin x$ **E)** none of these

17. Which of the following is an antiderivative of $-8x^3$?

A) $-\frac{8}{3}x^2$ **B)** $-2x^4$ **C)** $-24x^2$ **D)** $-24x^4$ **E)** none of these

18. Write the parametric equations $x(t) = 2 - 3t$ and $y(t) = -1 + 6t$ as $y = f(x)$.

A) $y = -2x + 3$ **B)** $y = -\frac{1}{2}x$ **C)** $y = -2x - 5$ **D)** $y = -\frac{1}{2}x - 2$ **E)** none of these

19. The product of two positive numbers is 300. Find the sum of the two numbers so that the sum of the first plus three times the second is as small as possible.

A) 30 **B)** 35 **C)** 40 **D)** 45 **E)** none of these

SCHOLARSHIP TEST 2017 ANSWER KEY

1. B
2. B
3. C
4. A
5. C
6. D
7. C
8. A
9. A
10. C
11. A
12. B
13. E
14. D
15. A
16. D
17. B
18. A
19. C