

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2018 MATH CONTEST
SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

1. Find the sum: $\sum_{k=0}^5 \frac{2^k}{\cos(\pi k)}$

- A) -21 B) -22 C) 10 D) 11 E) none of these

2. Which sequence has terms that approach zero as the sequence goes out to infinity?

- A) 24, 21, 18, 15, 12, ... B) $\frac{8^1}{1}, \frac{8^2}{2 \cdot 1}, \frac{8^3}{3 \cdot 2 \cdot 1}, \frac{8^4}{4 \cdot 3 \cdot 2 \cdot 1}, \dots$ C) 10, -9, 8, -7, 6, -5, 4, -3, 2, -1, ...
D) $\frac{1^3}{2 \cdot 1}, \frac{2^3}{3 \cdot 2}, \frac{3^3}{4 \cdot 3}, \frac{4^3}{5 \cdot 4}, \dots$ E) none of these

3. If $g(x) = \ln[(f(x))^2]$, $f(3) = 1$ and $f'(3) = 4$, find $g'(3)$.

- A) 0 B) $\frac{1}{2}$ C) 1 D) 8 E) none of these

4. If $f(x) = \sin(bx^c)$, find $f'(x)$.

- A) $bcx^{c-1}\cos(bx^c)$ B) $\cos(bcx^{c-1})$ C) $bx^{c-1}\cos(bx^c)$
D) $\cos(bx^c) + bcx^{c-1}\sin(bx^c)$ E) none of these

5. If e^{-25} is approximated by using the tangent line to the graph of $f(x) = e^{-5x}$ at (0,1), the approximation is?

- A) 1 B) 1.25 C) 1.28 D) $1 + e^{-25}$ E) none of these

6. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{36 - x^2}$. What is the maximum area the rectangle can have?

- A) $18\sqrt{2}$ B) 36 C) $36\sqrt{2}$ D) 18π E) none of these

7. If $\int_0^3 f(x)dx = 4$, $\int_{-2}^3 g(x)dx = 9$ and $\int_{-2}^0 g(x)dx = 4$, find $\int_0^3 (5f(x) - g(x)) dx$

- A) -45 B) 11 C) 15 D) 33 E) none of these

8. If $f(x)$ is an even function, $g(x)$ is an odd function, $\int_0^7 f(x)dx = 8$ and $\int_{-7}^0 g(x)dx = 4$, find $\int_{-7}^7 (f(x) + g(x))dx$.

- A) 8 B) 12 C) 16 D) 24 E) none of these

9. Evaluate $\int_0^4 |3x - 3|dx$.

- A) 12 B) 15 C) 18 D) 24 E) none of these

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10. Eliminate the parameter t . Find a rectangular equation for the plane curve defined by the parametric equations.

$$x = 2 + \sec(t) \quad y = 5 + 2 \tan(t); \quad 0 < t < \frac{\pi}{2}$$

- A) $(x - 2)^2 - (y - 5)^2 = 4 \quad -\infty < x < \infty$ B) $(x - 2)^2 + \frac{(y-5)^2}{4} = 1 \quad 1 \leq x \leq 3$
 C) $(x - 2)^2 - \frac{(y-5)^2}{4} = 1 \quad x > 3$ D) $(y - 2)^2 - \frac{(x-5)^2}{4} = 4 \quad -\infty < x < \infty$
 E) none of these

11. How many ordered pair solutions exist for the following system? $\frac{(y-1)^2}{9} - \frac{(x+4)^2}{16} = 1$ and $y = 3(x + 3)^2 - 2$

- A) 1 B) 2 C) 3 D) 4 E) none of these

12. Elly Smith makes floral arrangements. She has 15 different cut flowers and plans to use exactly 5 of them. How many different arrangements can she create?

- A) 75 B) 3003 C) 36,300 D) 360,360 E) none of these

13. In how many ways can the expression $x^5y^2z^5$ be rewritten as a product without exponents?

- A) 479,001,600 B) 9,580,032 C) 16,632 D) 9,256 E) none of these

14. A group of 30 students from your school attends a magic show, and the students are required to sit apart from one another throughout the auditorium. The total number of people in the audience is 150. The magician asks a total of 8 students to come up and help him with his act. If they are chosen randomly, what is the theoretical probability of 5 students from your school being selected? (Round the answer to the nearest 0.1%.)

- A) 16.7% B) 3.3% C) 1.2% D) 0.8% E) none of these

15. Given the definition of function g below, evaluate the limit: $\lim_{x \rightarrow 1} g(x)$

$$g(x) = \begin{cases} 4x + 2 & x > 1 \\ x^2 + 2 & x < 1 \\ 4 & x = 1 \end{cases}$$

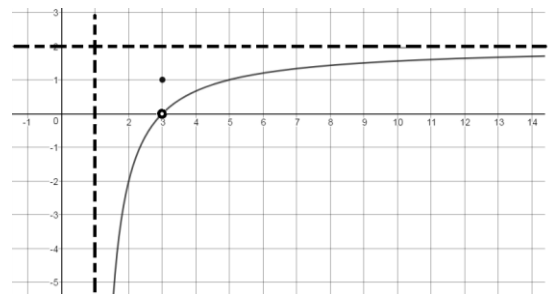
- A) 6 B) 4 C) 3 D) 1 E) none of these

16. Find all values of k that make the following limit equal to zero: $\lim_{x \rightarrow \infty} \frac{\sin(x) + \cos(x)}{x^k}$

- A) $k \geq 1$ B) $k > 1$ C) $k > 0$ D) $k \geq 0$ E) none of these

17. Which limit does NOT exist for the function $f(x)$ pictured?

- A) $\lim_{x \rightarrow 1} f(x)$ B) $\lim_{x \rightarrow 3^+} f(x)$
 C) $\lim_{x \rightarrow 5} f(x)$ D) $\lim_{x \rightarrow \infty} f(x)$
 E) none of these



18. At the beginning of **each** year, Sarah contributes \$500 to an investment account. The money invested in the account gains 4% by the end of each year. To the nearest dollar, how much will be in the account at the end of the 10th year?

- A) \$5503 B) \$6243 C) \$6743 D) \$7512 E) none of these

SCHOLARSHIP TEST 2018 ANSWER KEY

- 1. A**
- 2. B**
- 3. D**
- 4. A**
- 5. B**
- 6. B**
- 7. C**
- 8. C**
- 9. B**
- 10. C**
- 11. B**
- 12. B**
- 13. C**
- 14. D**
- 15. E**
- 16. C**
- 17. A**
- 18. B**