



MONTANA COUNCIL OF TEACHERS OF MATHEMATICS  
2018 MATH CONTEST  
SENIOR

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

1. Divide the complex numbers and express the result in standard (rectangular) form:  $\frac{9}{4-i}$
- A)  $\frac{12}{5} - \frac{3}{5}i$       B)  $\frac{36}{17} - \frac{9}{17}i$       C)  $\frac{12}{5} + \frac{3}{5}i$       D)  $\frac{36}{17} + \frac{9}{17}i$       E) none of these
2. Which parabola would have vertex (2, 3), focus (3, 3), and directrix  $x=1$ ?
- A)  $(y-3)^2 = 1(x-2)$       B)  $(y-3)^2 = 4(x-2)$       C)  $(y+3)^2 = -1(x+2)$   
D)  $(y-3)^2 = -4(x-2)$       E) none of these
3. An oval-shaped garden measures 46 feet across at its "longest span" and 26 feet across at its "shortest span." If a 6 foot wide walkway surrounds the garden, which equation below represents the conic section that most closely outlines the outside edge of the walkway?
- A)  $29^2x^2 + 19^2y^2 = 19^2 \cdot 29^2$       B)  $35^2x^2 + 25^2y^2 = 35^2 \cdot 25^2$       C)  $\frac{x^2}{46^2} + \frac{y^2}{26^2} = 6^2$   
D)  $\frac{x^2}{52^2} + \frac{y^2}{32^2} = 1$       E) none of these
4.  $a_1 = \sin(45^\circ) + \cos\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$ ,  $a_2 = \sin(90^\circ) + \cos(\pi) = 0$ ,  $a_3 = \sin(135^\circ) + \cos\left(\frac{3\pi}{2}\right) = \frac{\sqrt{2}}{2}$ ,  $a_4 = ?$
- A) 0      B)  $-\frac{\sqrt{2}}{2}$       C) 1      D)  $\frac{\sqrt{3}}{2}$       E) none of these
5. The **sum** of the diagonals and sides for a rectangle is 6. The **sum** of the diagonals and sides for a pentagon is 10. What is the **sum** of the diagonals and sides for a decagon?
- A) 21      B) 35      C) 45      D) 66      E) none of these
6. In factored form, a polynomial function is defined as  $f(x) = (x-2)(x+3)(x-k)$ , where  $k > 2$ . A local maximum must exist in what interval?
- A)  $x < -3$       B)  $-3 < x < 2$       C)  $2 < x < k$       D)  $x > 2$       E) none of these
7. There are two imaginary zeros for the function  $f(x) = 2x^4 - 17x^3 + 58x^2 - 77x + 26$ . If one of those imaginary zeros is  $3 + 2i$ , what is the **product** of its two imaginary zeros?
- A) 1      B) 5      C) 6      D) 13      E) none of these
8. Which polynomial function must have at least one local minimum for any values of a, b, or c?
- A)  $y = ax^3 + bx^2 + cx$       B)  $y = ax^3 + bx^2 + cx - 2x^4$       C)  $y = ax^6 + bx^4 + c$   
D)  $y = ax^6 + bx^4 + cx^2 + x^8$       E) none of these

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9. The greatest integer function can be denoted as  $y = [x]$ . For all domain values of  $x$ , it returns the largest integer less than or equal to  $x$ . Which “modified” greatest integer function has a range that only includes multiples of  $2k$ , where  $k$  is an integer?

- A)  $y = [x] + 2$       B)  $y = [2x]$       C)  $y = 2[x - 3]$       D)  $y = 2[x] + 1$       E) none of these

10. Using the vectors  $\vec{v} = 3\hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{w} = \hat{i} - 4\hat{j} + \hat{k}$ . Find  $\|2\vec{v} + \vec{w}\|$ .

- A) 6      B)  $5\sqrt{2}$       C)  $6\hat{i} + 0\hat{j} - \hat{k}$       D)  $8\hat{i} - 4\hat{j} + 0\hat{k}$       E) none of these

11. Find the resulting angle to the nearest degree between the two forces  $F_1 = 2\hat{i} - 4\hat{j}$  and  $F_2 = 7\hat{i} + 5\hat{j}$ .

- A)  $28^\circ$       B)  $81^\circ$       C)  $99^\circ$       D)  $101^\circ$       E) none of these

12. Find  $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{2x^2 - 4x - 6}$

- A)  $\frac{5}{4}$       B) 0      C)  $\frac{1}{2}$       D) does not exist      E) none of these

13. If  $\lim_{x \rightarrow c} f(x) = 4$  and  $\lim_{x \rightarrow c} g(x) = e$ , evaluate  $\lim_{x \rightarrow c} (\sqrt{f(x)} - \ln g(x))$

- A) 0      B) 1      C) 2      D)  $4 - e$       E) none of these

14. Find  $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 1}{e^{4x}}$

- A) 0      B)  $\frac{3}{2}$       C) 3      D)  $\infty$       E) none of these

15. When graphed for  $0 \leq t \leq 2\pi$ , the parametric equations  $x = 2 \sin(t) + 2$  and  $y = \cos(t) + 2$  results in:

- A) A line      B) A sinusoidal      C) A circle      D) An ellipse      E) none of these

16. The position of a projectile fired with an initial velocity  $v_0$  feet per second and at an angle  $\theta$  to the horizontal at the end of  $t$  seconds is given by the parametric equations  $x = (v_0 \cos \theta)t$  and  $y = (v_0 \sin(\theta))t - 16t^2$ . If  $v_0 = 100$  and  $\theta = 60^\circ$  for approximately what horizontal distance (to the nearest foot) is the projectile 20 feet or more above its original height?

- A) 5 feet      B) 52 feet      C) 196 feet      D) 246 feet      E) none of these

17. An open box of maximum volume is to be made from a square piece of material, 12 inches on each side, by cutting equal squares from each side. What is the maximum volume box you can make?

- A)  $100 \text{ in}^3$       B)  $122 \text{ in}^3$       C)  $128 \text{ in}^3$       D)  $156 \text{ in}^3$       E) none of these

18. Which is equivalent to  $(1 + i)^3$ ?

- A)  $1 - i$       B) 0      C)  $-2 + 2i$       D)  $-2 - 2i$       E) none of these

**SENIOR 2018 Answer Key**

1. D
2. B
3. A
4. C
5. C
6. B
7. D
8. D
9. C
10. B
11. C
12. A
13. B
14. A
15. D
16. D
17. C
18. C