

MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2019 MATH CONTEST
TEAM 11-12

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

For questions 1-3, use the graph of function f at right.

1. $\lim_{x \rightarrow 2^-} f(x) =$

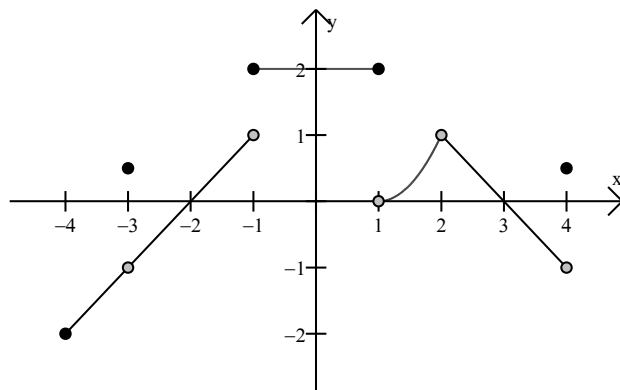
- A) 0 B) 1 C) 2
D) does not exist E) none of these

2. $\lim_{x \rightarrow 2^+} f(x) =$

- A) 0 B) 1 C) 2
D) does not exist E) none of these

3. $f(2) =$

- A) 0 B) 1 C) 2 D) does not exist E) none of these



4. Which famous mathematician is commonly credited with devising a numerical method to approximate solutions for differential equations?

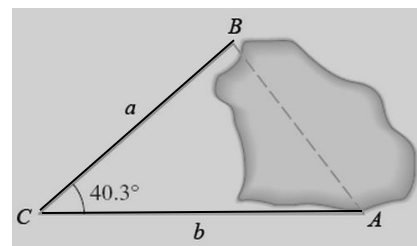
- A) Euler B) Leibniz C) Newton D) Riemann E) none of these

5. Find the horizontal asymptote(s) for the function $f(x) = \frac{3e^x + 5e^{2x}}{e^{2x} + 5}$.

- A) $y = 0$ B) $y = 3$ C) $y = 5$ D) $y = 0$ and $y = 5$ E) none of these

6. To find the distance across a large pond, a surveyor found the measurements shown at right. Find the distance AB when $a = 2.92$ miles, $b = 3.21$ miles and angle $C = 40.3^\circ$. Round your answer to the nearest hundredth.

- A) 2.08 mi B) 2.13 mi C) 4.53 mi D) 6.13 mi
E) none of these



7. On a particular Labor Day, the high tide in southern California occurs at 7:12am. At that time you measure the water at the end of the Santa Monica Pier to be 11 feet deep. At 1:24pm it is low tide, and you measure the water to be only 7 feet deep. Assume the depth of the water is a sinusoidal function of time with a period of $\frac{1}{2}$ a lunar day, which is about 12 hr 24 min. What is the *first* time on that Labor Day that the water reaches 9 feet deep?

- A) 4:06am B) 5:12am C) 10:18am D) 3:18pm E) none of these

8. Find the sum:

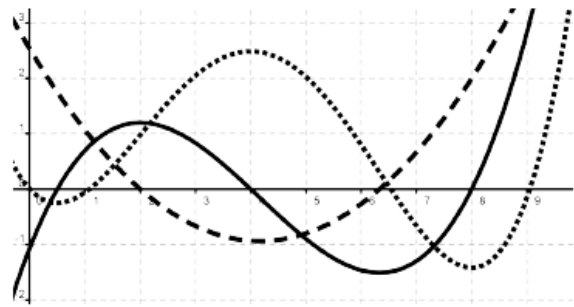
$$\sum_{k=1}^{\infty} 3 \left(\frac{3}{2}\right)^{k-1}$$

- A) -9 B) -6 C) 6 D) 9 E) none of these

9. A cylindrical can is to contain 900 cm^3 of liquid. What is the smallest possible surface area of the can? Round your answer to the nearest hundredth.

- A) 325.32 cm^2 B) 409.63 cm^2 C) 516.03 cm^2 D) 517.07 cm^2 E) none of these

10. The graphs of a function $f(x)$, its derivative $f'(x)$, and an antiderivative $\int f(x)dx$ is shown at right. Identify which combination is correct.



- A) dashed: $f(x)$, dotted: $f'(x)$, solid: $\int f(x)dx$
- B) dashed: $\int f(x)dx$, dotted: $f'(x)$, solid: $f(x)$
- C) dashed: $f'(x)$, dotted: $\int f(x)dx$, solid: $f(x)$
- D) dashed: $f(x)$, dotted: $\int f(x)dx$, solid: $f'(x)$
- E) none of these

11. Determine the number of vertical asymptotes for the function $f(x) = \frac{x^2 - 1}{x^4 + 27x^3 + 49x^2 - 27x - 50}$.

- A) 0
- B) 2
- C) 3
- D) 4
- E) none of these

12. Consider an 8th degree polynomial function. Which of the following situations CANNOT exist?

- A) eight distinct real zeros
- B) five distinct real zeros and three distinct non-real zeros
- C) four distinct real zeros, a double real zero, and two distinct non-real zeros
- D) eight distinct non-real zeros
- E) none of these

13. Ten randomly selected students from each grade in your high school were surveyed regarding prom themes. What type of sampling does this represent?

- A) convenience
- B) stratified
- C) systematic
- D) voluntary response
- E) none of these

14. Companies whose stocks are listed on the NASDAQ stock exchange have their company name represented by either four or five letters (repetition of letters is allowed). What is the maximum number of companies that can be listed on the NASDAQ?

- A) 234
- B) 8,252,400
- C) 12,338,352
- D) $\approx 5.43 \times 10^{12}$
- E) none of these

15. The dean of a major university claims that the mean time for students to earn a Master's degree is at most 3.8 years. If a hypothesis test is performed, how should you interpret a decision that fails to reject the null hypothesis?

- A) There is sufficient evidence to support the claim $\mu \leq 3.8$.
- B) There is sufficient evidence to reject the claim $\mu \leq 3.8$.
- C) There is not sufficient evidence to reject the claim $\mu \leq 3.8$.
- D) There is not sufficient evidence to support the claim $\mu \leq 3.8$.
- E) none of these

16. The first three terms of an arithmetic sequence are $2x - 3$, $5x - 11$, $3x + 1$ respectively. The n^{th} term of the sequence is 2009. What is n ?

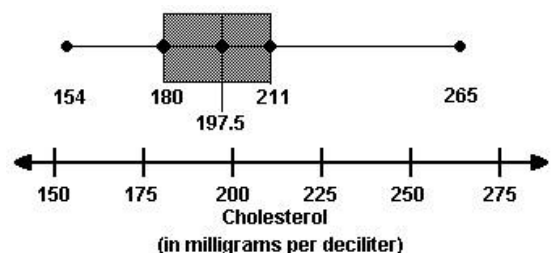
- A) 255
- B) 502
- C) 1004
- D) 1506
- E) none of these

17. Marcee is 20% older than Sheridan, and Sheridan is 40% younger than Devin. The sum of their ages is 23.2 years. How old will Marcee be on her next birthday?

- A) 7
- B) 8
- C) 9
- D) 10
- E) none of these

18. Which statement is accurate? Use the graph at right.

- A) One half of the cholesterol levels are between 180 and 211.
- B) About 75% of the adults have cholesterol levels less than 180.
- C) About 25% of the adults have cholesterol levels of at most 211.
- D) One half of the cholesterol levels are between 180 and 197.5.
- E) none of these



TEAM 11-12 2019 Answer Key

1. B
2. B
3. D
4. A
5. D
6. B
7. A
8. E
9. C
10. C
11. B
12. B
13. B
14. C
15. C
16. B
17. B
18. A