MONTANA COUNCIL OF TEACHERS OF MATHEMATICS
2019 MATH CONTEST
SCHOLARSHIP TEST

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

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1. Determine $\lim\limits_{x\to 2} \dfrac{x^3-12x^2+45x-50}{x-2}$.

A) –3 **B)** 2 **C)** 9 **D)** 25 **E)** none of these

2. Determine $\lim\limits_{x\to 3^+} \begin{cases} 2x-1 & if\ x \geq 3 \\ -2 & if\ 0 < x < 3 \\ x^2-3x & if\ x \leq 0 \end{cases}$.

A) –1 **B)** 0 **C)** 3 **D)** 5 **E)** none of these

3. Given $f(2) = 3$, $g(2) = -1$, $f'(2) = 2$, $g'(2) = 5$, then if $h(x) = f(x) \cdot g(x)$ determine $h'(2)$.

A) 0 **B)** 1 **C)** 10 **D)** 13 **E)** none of these

4. Given $f(2) = 8$, $g(2) = 2$, $f'(2) = -3$, $g'(2) = 3$, then if $m(x) = f(g(x))$ determine $m'(2)$.

A) –9 **B)** –6 **C)** –3 **D)** 16 **E)** none of these

5. Determine the equation of the line tangent to the curve $f(x) = \sqrt{2x-3}$ at $x = 2$.

A) $y = 0.5x - 2$ **B)** $y = 0.5x - 1$ **C)** $y = x - 1$ **D)** $y = x - 0.5$ **E)** none of these

6. If $\int_{-1}^{12} k(x)\,dx = -28$ and $\int_{3}^{12} k(x)\,dx = -10$, determine $\int_{-1}^{3} k(x)\,dx$.

A) –18 **B)** 11 **C)** 18 **D)** 38 **E)** none of these

7. If $\int_{2}^{7} p(x)\,dx = 4$, determine $\int_{7}^{2} (3p(x) + 8)\,dx$.

A) –104 **B)** –52 **C)** 20 **D)** 28 **E)** none of these

8. A class of 25 students has 15 girls and 10 boys. If the teacher randomly picks three students from the class what is the probability, to the nearest percent, the teacher will pick two boys and one girl?

A) 7% **B)** 10% **C)** 29% **D)** 33% **E)** none of these

9. In how many different ways can the nine starters on a baseball team be introduced if the pitcher is always introduced last and the catcher must be introduced first or second?

A) 1440 **B)** 5040 **C)** 10080 **D)** 40320 **E)** none of these

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10. Given the series $2 + 2.2 + 2.42 + 2.662 + \cdots$, what is the minimum number of terms needed for the sum to be greater than 81?

A) 17 **B)** 20 **C)** 21 **D)** 27 **E)** none of these

11. When a ball is dropped it bounces to 70% of its previous height. If it is dropped from 10 feet how far does it travel before it comes to rest?

A) 20 **B)** $33\frac{1}{3}$ **C)** $46\frac{2}{3}$ **D)** $56\frac{2}{3}$ **E)** none of these

12. Bob is making a pizza with three different toppings. He has 10 different toppings to choose from. How many different possibilities does Bob have?

A) 30 **B)** 120 **C)** 360 **D)** 720 **E)** none of these

13. What is the greatest possible area for a rectangle whose length is $15x - 2x^2$ and whose width is x?

A) 5 **B)** 7.5 **C)** 75 **D)** 125 **E)** none of these

14. A rectangular garden with an area of $200ft^2$ is to be fenced to protect vegetables from wild rabbits. Find the dimensions of the garden that will require the least amount of fencing if one side of the garden is protected by a barn.

A) 10 x 20 ft **B)** 8 x 20 ft **C)** 5 x 4200 ft **D)** 2 x 100 ft **E)** none of these

15. Write the parametric equations $y(t) = \sqrt{3t-4}$ and $x(t) = 6 - t$ as $y = f(x)$.

A) $f(x) = \sqrt{14-3x}$ **B)** $f(x) = \pm\sqrt{3x-14}$ **C)** $f(x) = \sqrt{3x-14}$
D) $f(x) = \pm\sqrt{14-3x}$ **E)** none of these

16. Find the focus of the function $x = \frac{1}{2}(y+2)^2 + 3$.

A) $(2.5\,, -2\,)$ **B)** $(3\,, -1.5\,)$ **C)** $(3\,, -2\,)$ **D)** $(3.5\,, -2\,)$ **E)** none of these

17. Find the equation of a circle if the endpoints of a diameter are $(-7, 1)$ and $(5,1)$.

A) $x^2 + (y-1)^2 = 144$ **B)** $(x+1)^2 + (y-1)^2 = 36$ **C)** $(x-1)^2 + y^2 = 36$
D) $(x-1)^2 + (y+1)^2 = 144$ **E)** none of these

18. Determine the maximum value of the function $P(x, y) = 3x^2 + 2y$, given that $x > 2$, $0 \le y \le 4$, and $x - y \le 6$.

A) 108 **B)** 208 **C)** 308 **D)** 408 **E)** none of these

19. Determine $\lim_{x\to-\infty} \frac{\sqrt{4x^2-5x+3}}{5x}$.

A) $-\infty$ **B)** $-\frac{1}{5}$ **C)** $\frac{2}{5}$ **D)** $\frac{4}{5}$ **E)** none of these

SCHOLARSHIP TEST 2019 ANSWER KEY

1. C
2. D
3. D
4. A
5. C
6. A
7. B
8. C
9. C
10. A
11. D
12. B
13. D
14. A
15. A
16. D
17. B
18. C
19. C