



MONTANA COUNCIL OF TEACHERS OF MATHEMATICS  
2019 MATH CONTEST  
SENIOR

DIRECTIONS: DO NOT WRITE ON THIS TEST. Place the best answer for each question on the separate answer sheet.

1. Determine  $\lim_{x \rightarrow \infty} \frac{5-2x+3x^3}{4x^3-1}$ .
- A) 0                      B) 0.75                      C) 1.25                      D)  $\infty$                       E) none of these
2. What  $a$ -value makes  $f(x) = \begin{cases} ax^2 + 6 & \text{if } x \leq -1 \\ ax - 4 & \text{if } x > -1 \end{cases}$  a continuous function?
- A) -10                      B) -6                      C) -5                      D) -4                      E) none of these
3. Given the greatest integer function  $f(x) = \frac{1}{2} \llbracket 1 - 2x \rrbracket$  which function values are different?
- A)  $f(-0.5), f(0)$                       B)  $f(0.9), f(1)$                       C)  $f(1.8), f(2)$                       D)  $f(4.7), f(5)$                       E) none of these
4. If  $b(x)$  has a degree of 2 and  $e(x)$  has a degree of 5 determine the degree of  $b(e(x)) + e(b(x))$ .
- A) 7                      B) 9                      C) 10                      D) 20                      E) none of these
5. An open box is formed from a 40 cm x 50 cm piece of thick paper after cutting squares of length  $x$  from each corner and folding up the edges creating a box of height  $x$  cm. Determine the height of the box which creates the maximum possible volume. Round the answer to the nearest tenth of a centimeter.
- A) 7.0 cm                      B) 7.4 cm                      C) 7.8 cm                      D) 8.2 cm                      E) none of these
6. Determine  $\lim_{x \rightarrow 2} \frac{x+3}{2-x}$ .
- A)  $-\infty$                       B) -1                      C)  $\frac{3}{2}$                       D)  $\infty$                       E) does not exist
7. Consider the vectors  $\langle 3, -7, 4 \rangle$  and  $\langle -4, -2, 1 \rangle$ . Determine the dot product of the vectors and state if they are orthogonal.
- A) 6, orthogonal                      B) 6, not orthogonal                      C) 22, orthogonal                      D) 22, not orthogonal                      E) none of these
8. Determine the resulting velocity of a boat traveling south at 4.5 m/s that encounters a current traveling 3.0 m/s  $50^\circ$  south of west. Round all answers to the nearest tenth.
- A) 7.1 m/s @  $15.8^\circ$  W of S                      B) 7.5 m/s @  $24.2^\circ$  S of W                      C) 710 m/s @  $24.2^\circ$  W of S  
D) 750 m/s @  $15.8^\circ$  S of W                      E) none of these
9. Which of the following equations represents parabola that opens to the left in Standard form?
- A)  $-\frac{1}{2}y^2 + 6 = x$                       B)  $2y = x^2 + 12$                       C)  $2x + y^2 = 12$                       D)  $2x^2 + y^2 = 12$                       E) none of these

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10. Which is the equation of an ellipse with center  $(1, -2)$  and a vertical major axis?

- A)  $\frac{(y-2)^2}{9} + \frac{(x+1)^2}{4} = 1$       B)  $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$       C)  $\frac{(y+2)^2}{9} + \frac{(x-1)^2}{4} = 1$   
D)  $\frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1$       E) none of these

11. When graphed, the parametric equations  $x = 8 \cos(t)$  and  $y = \sin(t)$  results in:

- A) an ellipse      B) a circle      C) a sinusoidal      D) a line      E) none of these

12. How long is the vector whose tail is at the point  $(-1, 4, 2)$  and whose tip at the point  $(3, 2, 3)$ ?

- A) 21      B)  $\sqrt{21}$       C) 9      D)  $\sqrt{9}$       E) none of these

13. Simplify the complex expression and write the answer in standard (rectangular) form:  $\frac{(1+2i)^3}{2i}$

- A)  $-1 + 5.5i$       B)  $4 + 0.5i$       C)  $1 + 8i$       D)  $2 + 1.5i$       E) none of these

14. Solve  $(2 + i)(x + 3i) = 3 + 9i$  for  $x$ .

- A)  $-3$       B) 1      C) 2      D) 3      E) none of these

15. What is the next term in the sequence 1, 4, 6, 14, 26 ?

- A) 40      B) 52      C) 54      D) 56      E) none of these

16. What is the 7<sup>th</sup> term in the sequence  $-2, 5, 24, 61, 122$  ?

- A) 244      B) 340      C) 345      D) 366      E) none of these

17. What is the minimum distance between the line  $y = -x + 4$  and the origin?

- A) 2      B)  $2\sqrt{2}$       C) 4      D)  $2\sqrt{3}$       E) none of these

18. A baseball is modelled by the parametric equations  $y = -16t^2 + 39t + 3$  and  $x = 92t$ . How high is the ball when it has travelled horizontally 200 feet? Round to the nearest tenth of foot.

- A) 0 ft      B) 2.2 ft      C) 12.2 ft      D) 15.2 ft      E) none of these

19. Determine  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$ .

- A) 0      B) 1      C) 5      D)  $\infty$       E) does not exist

20. How many x-intercepts exist for the function  $f(x) = x(x - 12)^3(2x - 30)^2(x + 5)$ ?

- A) 2      B) 3      C) 4      D) 7      E) none of these

## SENIOR 2019 Answer Key

1. B
2. C
3. A
4. C
5. B
6. E
7. B
8. A
9. C
10. C
11. A
12. B
13. A
14. D
15. C
16. B
17. B
18. C
19. C
20. C