Proving the Pythagorean Theorem
A Geometry Activity for Middle School
by Hilary Smith Risser

**Objectives:** This activity addresses the following CCSS-M standards from 8th grade.

8.EE.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions
8.EE.7b: Solve linear equations with rational number coefficients including equations whose solutions require expanding expressions using the distributive property and collecting like terms
8.G.6: Explain a proof of the Pythagorean Theorem and its converse

**Lesson Description:** In this activity, students will prove the Pythagorean Theorem and its converse.

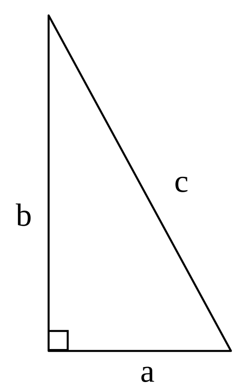
*Pythagorean Theorem:*
Given a triangle with sides of length *a*, *b*, and *c* where *c* is the longest side, if the triangle is a right triangle then $a^2 + b^2 = c^2$

*Converse of the Pythagorean Theorem:*
Given a triangle with sides of length *a*, *b*, and *c* where *c* is the longest side, if $a^2 + b^2 = c^2$ then the triangle is a right triangle with the right angle opposite the longest side

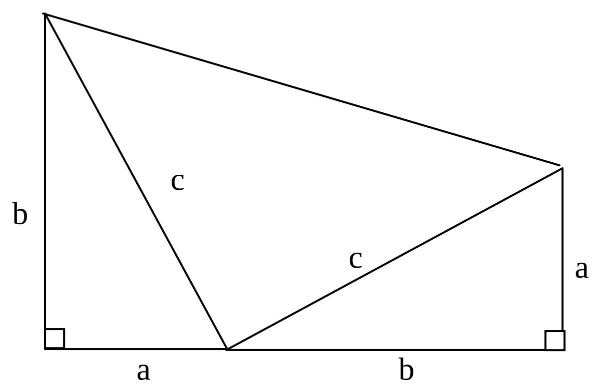
Students will repeat a proof first published by President James Garfield to prove the Pythagorean Theorem

*Garfield's Proof:*

The proof begins with a right triangle.




Two copies of this triangle are used to create a composite figure



1. Have students begin the activity by finding the area of the composite figure. Students could use the formula for area of a right trapezoid or they could decompose the figure into a rectangle and a right triangle. The expression they end up with should be algebraically equivalent to $\frac{1}{2}\left(a^2+2ab+b^2\right)$.

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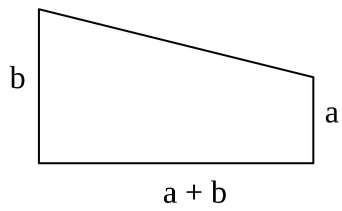
<td>
</td>
<td>$A=\frac{1}{2}(a+b)(a+b)$</td>
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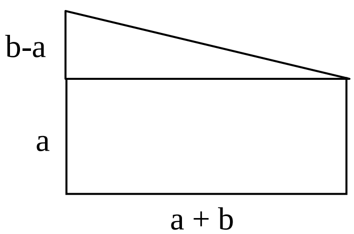
<td>
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<td>$A=a(a+b)+\frac{1}{2}(b-a)(a+b)$</td>
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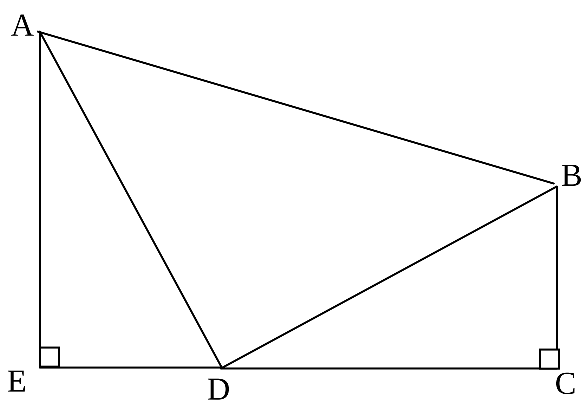
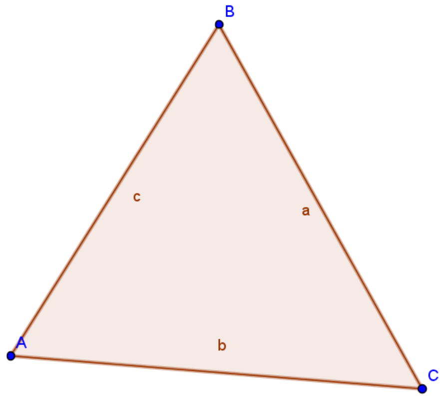
2. Show the students Garfield's picture. They are going to find the area of each of the three right triangles. Ask students what they would need to know to find the area of triangle ABD. They should realize that they will need to know the measure of angle ADB. They can use the congruent triangles AED and DCB to find that ADB is a right angle.



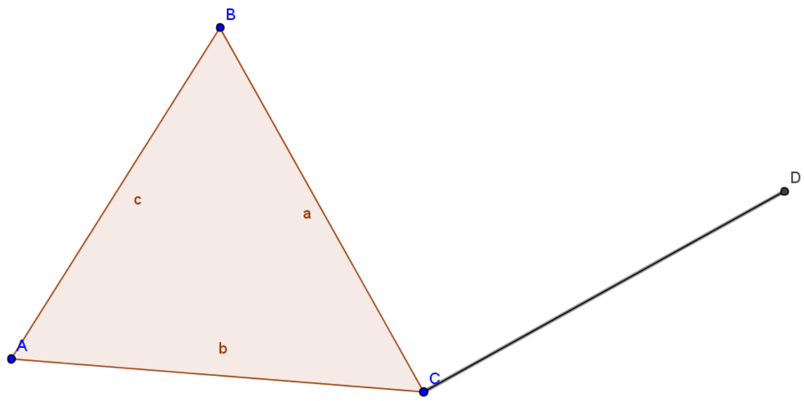

3. Have students use area formulas to find the area for each of the three right triangles. They should find an expression of the form $\frac{1}{2}ab+\frac{1}{2}ab+\frac{1}{2}c^2$

4. Have students set the formula they found in #1 and in #3 equal to one another and solve the expression for $c^2$. The result will be the Pythagorean Theorem.

For the demonstration of the converse create a triangle with sides *a*, *b*, and *c*. We will assume that $a^2 + b^2 = c^2$. However, we will not assume that the triangle is a right triangle. In order to prevent students from assuming that the triangle is a right triangle, draw ABC so that it does not look like a right triangle.




Then construct a line segment CD. This line segment should be perpendicular to BC and be congruent to AC.




By connecting points B and D with a line segment, we have now produced a right triangle: triangle BCD. Since we know the Pythagorean theorem applies to right triangles, we also know that

$$CD^2 + BC^2 = BD^2$$

Now, we have constructed CD to be congruent to AC and BC is congruent to itself, so our equation becomes

$$a^2 + b^2 = BD^2$$

From our original assumption, we know that $a^2 + b^2 = c^2$. This means that

$$c^2 = BD^2$$

or that BD must be congruent to *c*.  So we now know that triangles ABC and DCB are congruent.  This means that ABC must also be a right triangle.

The converse of the Pythagorean theorem allows us to use the three side lengths of a triangle to determine if the triangle is right, acute, or obtuse.  The converse of the Pythagorean Theorem can be proven without using the Pythagorean Theorem.  However, this proof requires students to use the Law of Cosines and is therefore not appropriate for middle school students.