VISUALIZING SIMILARITIES IN RIGHT TRIANGLES USING 3 X 5 CARDS

CCSS: Geometry G-SRT

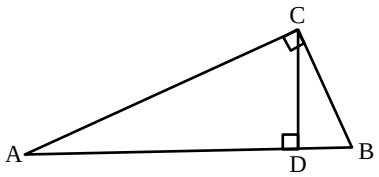
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Prerequisites: Knowledge of geometric means, similar triangles, ability to measure lengths and angles

Materials: 3 x 5 cards or any other rectangular piece of paper. (4 x 6 card, 8 ½ x 11 paper, etc.)
Straight edges, and scissors

When using theorems involving the altitude to the hypotenuse of a right triangle, my students have difficulties aligning the corresponding vertices of the similar triangles and picking out the geometric means involved. The theorems in question are:

1) The altitude to the hypotenuse of a right triangle produces two right triangles similar to each other and the original triangle.
2) The altitude to the hypotenuse of a right triangle is the geometric mean between the segments of the hypotenuse.
3) When the altitude to the hypotenuse of a right triangle is drawn, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to the leg.



1) $\Delta ABC \sim \Delta ACD \sim \Delta CBD$

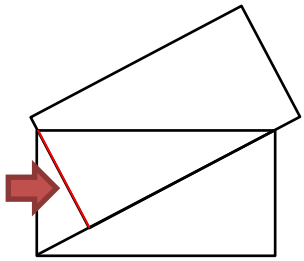
2) $\frac{AD}{CD} = \frac{CD}{DB}$

3) $\frac{AD}{AC} = \frac{AC}{AB}$ and $\frac{DB}{BC} = \frac{BC}{AB}$

I use a 3 x 5 card (or any rectangle) to help them visualize the relationships.

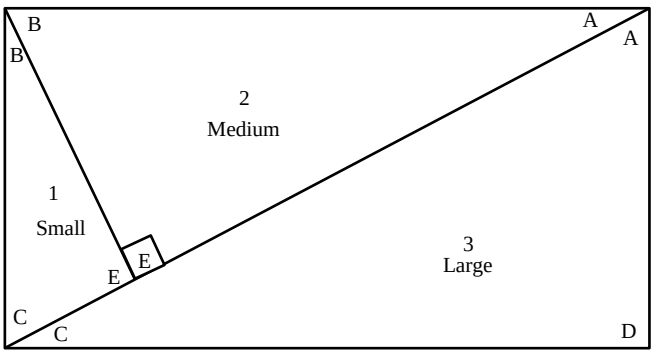
1) Draw a diagonal on the card.

2) Using a partner's card. Lay the long side of the card on the diagonal and position it so the short side of the card passed through the corner of the card. (See diagram at the right)

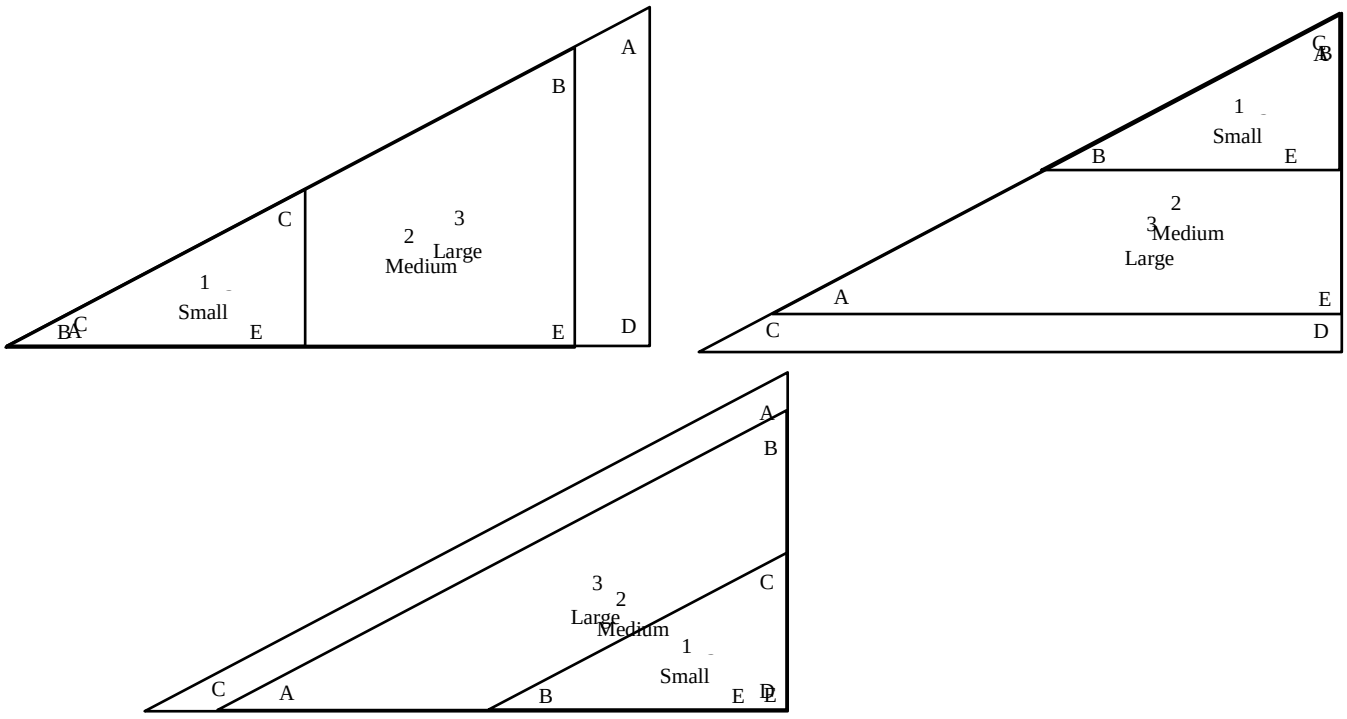


3) Draw the altitude. (the red line)

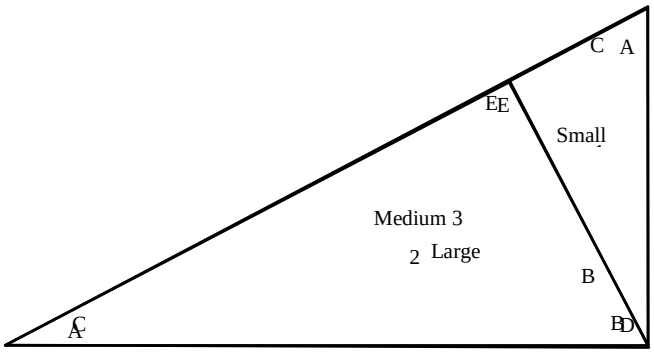
4) Label the triangles as in the diagram on both sides of the card.



5) Cut out the triangles. Place them on top of each other, matching the congruent angles. There are three different ways to do this. You can match the small angles, or the medium angles or the right angles.



6) Place the two smaller triangles on top of the large triangle to show the altitude to the hypotenuse of a right triangle.



7) You can measure all of the sides and angles and set up the ratios of the sides to show that the triangles are similar. These can also be used to find the sine, cosine and tangent ratios.

8) After the triangles are shown to be similar, the geometric means can be set up. Then I have the students set up the geometric means in several different orientations of the triangles. We usually do an assignment out of the book, followed by a worksheet or two, depending on how much the class needs to gain proficiency.

After measuring all of the sides in millimeters and the angles in degrees, these same triangles can be used to find sine, cosine, and tangent ratios of the acute angles. The Pythagorean Theorem can be demonstrated with them; and who knows what else they can be used for. I really like using visuals and manipulatives to demonstrate geometric ideas. I got this idea from the February 1996 edition of the Mathematics Teacher in an article entitled “The Incredible Three-by-Five Card!” by Dan Lufkin.